

Interaction of Two Reggeized Gluons: 2nd Order Corrections and the Bootstrap

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Our aim is to show how the reggeization of the gluon, encoded in the bootstrap property of the BFKL kernel, permits to calculate the interaction kernel in the octet colour channel in the forward and non forward direction, for the quark contribution, starting from the known gluon trajectory up to 2nd order. To this end an ansatz to solve the bootstrap equation is used. The obtained result, for what regard the quark contribution, has been verified by direct Feynman diagram calculations by Fadin et al..

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1 Introduction

The BFKL equation and its impact in small x physics have been widely discussed subjects in the last years and recently the 2nd order corrections were calculated for the forward pomeron equation^{1,2}. Generalization of this result in the non forward direction would, of course, be really interesting, not only in the singlet colour channel of the pomeron. For the other physically important case, that of the odderon, the 2nd order corrections to the non-forward interaction of two reggeized gluons in the octet colour state is needed (as well as also the form of the gluon triple interaction term).

This interaction is severely restricted by the so-called bootstrap relation which guarantees that production amplitudes with the gluon quantum number in their t channels used for the construction of the absorptive part are indeed given only by a single reggeized gluon exchange and do not contain admixture from two or more reggeized gluon exchanges^{3,4}. The bootstrap relation is known to be satisfied in the lowest order in the coupling constant. The form this relation takes in the 2nd order was recently discussed by Fadin et al.⁵

The bootstrap can be expressed as a relation which involve the reggeized gluon trajectory, the interaction between two reggeized gluons and their wave function. We show here⁶ how, using the bootstrap in a form stronger than in⁵ and an ansatz⁷ to solve it, it is possible to obtain the full second order corrections, at least due to the quark contribution, for the non-forward gluon

interaction in the octet channel necessary for the odderon equation. It can also be shown that we also automatically find the corrections for the running of the coupling for the non-forward pomeron equation⁶.

2 The Bootstrap relation

We find convenient to work with an operator formalism to describe the two gluon equation and the bootstrap relation. Let the two gluons with momenta $q_{1,2}$ be described by a wave function $\Psi(q_1, q_2)$. The total momentum $q = q_1 + q_2$ will always be conserved, so that in future $q_2 = q - q_1$ and the dependence on q and thus q_2 will be suppressed. The scalar product of two wave functions will be taken as $\langle \Psi_1 | \Psi_2 \rangle \equiv \int d^{D-2} q_1 \Psi_1^*(q_1) \Psi_2(q_1)$, where $D = 4 + 2\epsilon$, $\epsilon \rightarrow 0$, is the dimension used for the infrared regularization.

In this metric, as a function of the angular momentum j and up to order α_s^3 , the absorptive part of the scattering amplitude coming from the two-reggeized-gluon exchange is given by $\mathcal{A}_j = \langle \Phi_p | G(E) | \Phi_t \rangle$.

Here the functions $\Phi_{p,t}$ are the so called impact factors which represent the coupling of the external particles (t from the target and p from the projectile) to the two exchanged gluons and are of order $\alpha_s = g^2/4\pi$.

The Green function $G(E)$ with $E = 1 - j$ is defined as $G(E) = (H - E)^{-1}$ where H is the two gluon (Hermithean) Hamiltonian. Its explicit expression in the momentum representation is

$$\langle q_1 | H | q'_1 \rangle = -\delta^{D-2}(q_1 - q'_1)(\omega(q_1) + \omega(q_2)) - V^{(R)}(q_1, q'_1) \quad (1)$$

where $1 + \omega(q_{1(2)})$ is the Regge trajectory of the first (second) gluon and $V^{(R)}$ represents the gluonic interaction for the t -channel with the colour quantum number R .

In any colour channel the Green function $G(E)$ can be represented via the orthonormalized solutions of the homogeneous Schrödinger equation $H\Psi_n = E_n\Psi_n$. Therefore, in terms of the eigenfunctions Ψ_n the absorptive part \mathcal{A}_j is

$$\mathcal{A}_j = \sum_n \frac{\langle \Phi_p | \Psi_n \rangle \langle \Psi_n | \Phi_t \rangle}{E_n - E}. \quad (2)$$

Let us consider the case with a colour quantum number of the gluon ($R = g$) in the t channel. The BFKL equation is based on the idea that physical amplitudes in this channel have the asymptotical behaviour corresponding to an exchange of a single reggeized gluon so that no contribution should come from the exchange of two or more reggeized gluons (in the leading (LO) and next-to-leading (NLO) orders). This means that the absorptive part, as a

function j , should have a simple pole at $j = 1 + \omega(q)$ and no other singularities. We can therefore state the following properties valid in the **LO and NLO**:

1. the spectrum of H in the considered t channel should contain an eigenvalue $E_0 = -\omega(q)$ with the corresponding eigenfunction Ψ_0

$$(H + \omega(q))\Psi_0 = 0 \quad (3)$$

2. the expression in (2) take contributions only from this eigenfunctions. This means that both impact factors $\Phi_{p,t}$ should be orthogonal to all other eigenfunctions Ψ_n , $n > 0$. Due to the completeness of the whole set it means that both Φ_p and Φ_t should coincide with Ψ_0 , up to a normalization factor, which will contain all the dependencies in the other variables (factorization).

3. there is a relation for the impact factor. In fact one considers the Mellin transform for the amplitude in (2) where only the contribution for $n = 0$ is present in the sum. Taking into account the proper signature factor and comparing to the standard form, where the coupling Γ 's (particle-particle-reggeon vertices) are factorized from the contribution of the reggeized gluon, one obtains the “third bootstrap condition”

$$\Gamma(q) = q \left(\frac{q^2}{s_0} \right)^{\omega(q)/2} \frac{\langle \Phi | \Psi_0 \rangle}{\sqrt{|\sin \pi \omega(q)|}}. \quad (4)$$

Considering a perturbative approach one may write $\Psi_0 = \chi + \Psi_0^{(1)}$, $\omega(q) = \omega^{(1)}(q) + \omega^{(2)}(q)$ and $H = H^{(1)} + H^{(2)}$ and obtain at the second order for the bootstrap eq. (3) the form

$$(H^{(1)} + \omega^{(1)}(q))\Psi_0^{(1)} = -(H^{(2)} + \omega^{(2)}(q))\chi \quad (5)$$

This general relation may be projected onto the state χ (the first order wave function) to obtain the relation

$$\langle \chi | H^{(2)} + \omega^{(2)}(q) | \chi \rangle = 0 \quad (6)$$

which has been previously obtained⁵ from different arguments together with the second order form of (4). We shall denote the relation (6) as a weak bootstrap condition.

3 Bootstrap at work

The main idea consists in taking one particular solution of the bootstrap relation, which describe all the inter-dependent quantities in terms of a single

function. Explicitly in the LO BFKL equation every transverse momentum squares are substituted by a function of them which we call η . Explicitly taking:

$$\omega(q) = - \int d^2 q_1 \frac{\eta(q)}{\eta(q_1)\eta(q_2)} \quad (7)$$

and

$$V^{(g)}(q_1, q'_1) = \frac{1}{\sqrt{\eta(q_1)\eta(q_2)\eta(q'_1)\eta(q'_2)}} \left(\frac{\eta(q_1)\eta(q'_2) + \eta(q_2)\eta(q'_1)}{\eta(k)} - \eta(q) \right) \quad (8)$$

then Eq.(3) remains valid with a solution

$$\Psi_0 = 1/\sqrt{\eta(q_1)\eta(q_2)} \quad (9)$$

Therefore the strategy to determine the interaction potential for the two gluons in the octet state will be the following: known gluon trajectory at NLO \rightarrow determination of $\eta \rightarrow$ construction of the interaction. In the LO case one has $\eta^{(0)}(q) = q^2/a$, where $a = \frac{q^2 N}{2(2\pi)^{D-1}}$, and in general we shall assume at the NLO the form $\eta(q) = \eta^{(0)}(q)(1 + \xi(q))$.

We briefly describe the calculations leading to the full interaction at NLO for the octet state⁶. We first consider the expressions for the trajectory:

$$\omega^{(1)}(q) = -\bar{g}^2 q^{2\epsilon} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} \quad , \quad \omega^{(2)}(q) = -\bar{g}^4 q^{4\epsilon} F(\epsilon) \quad (10)$$

where

$$\bar{g}^2 = \frac{Ng^2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \quad , \quad F(\epsilon) = \frac{A}{\epsilon^2} - \frac{B}{\epsilon} - C - \frac{\pi^2}{3}A \quad (11)$$

The coefficients A, B, C contains a part proportional to N_f (the number of quark flavours) which therefore isolate the quark contribution to the reggeized gluon trajectory $\omega_Q^{(2)}(q)$, proportional on $F_Q(\epsilon)$. Thus the quark contribution to η or to the ξ functions is fixed looking only at $\omega_Q^{(2)}(q)$. Let us write this ξ_Q . Using an integral representation for $\omega_Q^{(2)}(q)$ one can write

$$2bq^2 \int \frac{d^{D-2}q_1}{q_1^2 q_2^2} (q^{2\epsilon} - q_1^{2\epsilon} - q_2^{2\epsilon}) = -aq^2 \int \frac{d^{D-2}q_1}{q_1^2 q_2^2} [\xi(q) - \xi(q_1) - \xi(q_2)] \quad (12)$$

where the $\omega_Q^{(2)}(q)$ on the LHS is matched to the form (7) which solves the bootstrap equation and expanded to the second order; b is a function of the parameter ϵ with a pole in zero.

The last relation is solved for $\xi_Q(q) = -2bq^{2\epsilon}/a$ and substituting this in the second order part of the potential (8) one easily obtains, for the quark contribution,

$$V_Q^{(g,2)}(q_1, q'_1) = -b \frac{1}{\sqrt{q_1^2 q_2^2 q_1'^2 q_2'^2}} \left[\frac{q_1^2 q_2'^2}{k^2} (q_1^{2\epsilon} + q_2'^{2\epsilon} - q_2^{2\epsilon} - q_1'^{2\epsilon} - 2k^{2\epsilon}) + \right. \\ \left. \frac{q_2^2 q_1'^2}{k^2} (q_2^{2\epsilon} + q_1'^{2\epsilon} - q_1^{2\epsilon} - q_2'^{2\epsilon} - 2k^{2\epsilon}) - q^2 (2q^{2\epsilon} - q_1^{2\epsilon} - q_2^{2\epsilon} - q_1'^{2\epsilon} - q_2'^{2\epsilon}) \right] \quad (13)$$

This expression (multiplied by $\sqrt{q_1^2 q_2^2 q_1'^2 q_2'^2}$ to match the integration measure) coincides with the one found later in⁸ by direct methods (see Eq. (47) there).

We try to use this procedure to calculate the total gluon interaction, with also the gluonic contribution. From the relations $\xi(q)/\xi_Q(q) = \omega^{(2)}(q)/\omega_Q^{(2)}(q) = F(\epsilon)/F_Q(\epsilon)$ the answer is

$$V^{(g,2)}(q_1, q'_1) = V_Q^{(g,2)}(q_1, q'_1) F(\epsilon)/F_Q(\epsilon) \quad (14)$$

Since the full kernel K , which solves the bootstrap relation, can be written in an explicitly infrared finite way it is convenient to write the expressions using the renormalized coupling constant g_μ . In the dimensional regularization in the $\overline{\text{MS}}$ scheme one has $g^2 = g_\mu^2 \mu^{-2\epsilon} (1 + A\bar{g}_\mu^2/\epsilon)$ and $\bar{g}_\mu^2 = g_\mu^2 N\Gamma(1 - \epsilon)/(4\pi)^{2+\epsilon}$. This means that we can also renormalize the η or the ξ functions. One has a finite $\xi_\mu(q) = \xi(q) - A\bar{g}_\mu^2/\epsilon$ because the poles in $1/\epsilon$ cancel. Therefore we are allowed to put $\epsilon = 0$ in the infrared finite expression for the non forward kernel in the octet channel. The simple form we present here refers to a metric modified in accord to $\Psi(q_1) \rightarrow \Psi(q_1)/\sqrt{\eta(q_1)\eta(q_2)}$. For the second order contribution (as usual a superscript indicates the order) we obtain

$$K^{(2)}\Psi = \frac{3\alpha_s}{4\pi} \left[BK^{(1)}\Psi + 6A\alpha_s \int \frac{d^2 q'_1 q_1^2}{(2\pi)^2 k^2} \left(\frac{\Psi(q'_1)}{q_1'^2} - \frac{\Psi(q_1)}{q_1'^2 + k^2} \right) \ln \frac{q_1^2 \mu^2}{q_1'^2 k^2} \right. \\ \left. + (1 \leftrightarrow 2) - 6A\alpha_s \int \frac{d^2 q'_1 q^2 \Psi(q'_1)}{(2\pi)^2 q_1'^2 q_2'^2} \ln \frac{q^2 \mu^2}{q_1'^2 q_2'^2} \right]. \quad (15)$$

Let us remark that from some recent calculations⁹ devoted to analyze the NLO quark and gluon impact factors calculated in¹⁰ we see that the ansatz given in (7-9) is correctly working for the $q\bar{q}$ pair but not for the gluon contribution even if the running of the coupling is correctly reproduced.

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